Name: _____

ID #: _____

Midterm 1

1. Compute

$$\int \frac{x^2}{\sqrt{9-x^2}}\,\mathrm{d}x.$$

2. Find the area of the surface obtained by rotating the curve given by

$$y = 2\sqrt{x+1}, \quad 0 \leq x \leq 3$$

around the $x$-axis.

3. Find $k \in \mathbb{R}$ such that

$$f(x) = \frac{k}{1+x^2}$$

defines a probability density function.

4. Compute

$$\int \frac{\mathrm{d}x}{1+\sqrt[3]{x}}.$$

5. For each of the following improper integrals decide whether it converges or diverges. No justification is needed.
(Correct answer = +3 points, wrong answer = 0 points, blank = 1.5 points)

| Integral | Converges | Diverges |
|---|---|---|
| (a) $\displaystyle\int_0^\infty x^{20}e^{-7x}\,\mathrm{d}x$ | | |
| (b) $\displaystyle\int_{-1}^1 \frac{1}{x}\,\mathrm{d}x$ | | |
| (c) $\displaystyle\int_{-\infty}^\infty \frac{e^{-x}}{1+x^2}\,\mathrm{d}x$ | | |
| (d) $\displaystyle\int_0^\infty \frac{e^{x}}{1+e^{2x}}\,\mathrm{d}x$ | | |
| (e) $\displaystyle\int_0^1 \frac{1+x^2}{1-x^2}\,\mathrm{d}x$ | | |
| (f) $\displaystyle\int_1^\infty \frac{x}{1+x^3}\,\mathrm{d}x$ | | |
| (g) $\displaystyle\int_1^\infty \frac{\arctan\left(\frac{1}{x}\right)}{x}\,\mathrm{d}x$ | | |